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14. ABSTRACT
An algorithm for calculating univariate L1 spline fits that involves multiple minimizations of the spline functional on each iteration was created. The computational results for this algorithm indicate overall good performance of the procedure but the procedure is computationally more expensive than desired. We formulated a new potential algorithm in which there will be only one minimization of the spline functional on each iteration. We also continued development of a new L1 "Multiple Component Detection and Analysis" (L1 MCDA) algorithm, which

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Report Title

L1 Splines with Locally Computed Coefficients

ABSTRACT

An algorithm for calculating univariate L1 spline fits that involves multiple minimizations of the spline functional on each iteration was created. The computational results for this algorithm indicate overall good performance of the procedure but the procedure is computationally more expensive than desired. We formulated a new potential algorithm in which there will be only one minimization of the spline functional on each iteration. We also continued development of a new L1 "Multiple Component Detection and Analysis" (L1 MCDA) algorithm, which is a fundamental and complete reformulation of Principal Component Analysis in a framework exclusively based on the L1 norm. Direct connection with heavy-tailed statistics is a guiding principle. We completed design of and computational results for the 2D case. The extension of L1 MCDA to 3D is currently under way.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
07/21/2011	1.00	John E. Lavery, Qingwei Jin, Shu-Cherng Fang. Univariate Cubic L1 Interpolating Splines: Analytical Results for Linearity, Convexity and Oscillation on 5-PointWindows, Algorithms, (07 2010): 0. doi: 10.3390/a3030276
07/21/2011	2.00	Lu Yu, Qingwei Jin, John E. Lavery, Shu-Cherng Fang. Univariate Cubic L1 Interpolating Splines: Spline Functional, Window Size and Analysis-based Algorithm, Algorithms, (08 2010): 0. doi: 10.3390/a3030311

TOTAL: 2

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

IIE 61st Annual Conference and Expo 2011, Reno, NV May 21-25, 2011
Title: Data Plotting Using Local L_1 Interpolating Splines
Presenter: Qingwei Jin
Authors: Jin, Q.; Tian, Y.; Lavery, J.E.; Fang, S.-C.

IIE 61st Annual Conference and Expo 2011, Reno, NV May 21-25, 2011
Title: Calculating Shape Preserving L_1 Splines for Data Interpolation
Presenter: Lu Yu
Authors: Yu, L.; Deng, Z.

Number of Presentations: 2.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

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Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received		Paper
01/08/2013	5.00	John E. Lavery. Univariate Lp and lp Averaging, 0 < p < 1, in Polynomial Time by Utilization of Statistical Structure, Algorithms (07 2012)
01/08/2013	6.00	Ye Tian, Qingwei Jin, John E. Lavery, Shu-Cherng Fang1. L1 Major Component Detection and Analysis (`1 MCDA): Foundations in Two Dimensions, Algorithms (10 2012)
08/19/2011	4.00	Qingwei Jin, Lu Yu, John E. Lavery, Shu-Cherng Fang. Univariate cubic L1 interpolating splines basedon the first derivative and on 5-point windows: Analysis, algorithm and shape-preserving properties, Computational Optimization and Applications (08 2011)

TOTAL: **3**

Number of Manuscripts:

Books

Received Paper

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Ye Tian	1.00	
Zhibin Deng	1.00	
FTE Equivalent:	**2.00**	
Total Number:	**2**	

Names of Post Doctorates

NAME PERCENT_SUPPORTED

FTE Equivalent:
Total Number:

Names of Faculty Supported

NAME PERCENT_SUPPORTED

FTE Equivalent:
Total Number:

Names of Under Graduate students supported

NAME PERCENT_SUPPORTED

FTE Equivalent:
Total Number:

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME
Ye Tian

Total Number: 1

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

The research was on L1 splines and L1 MCDA (Multiple Component Detection and Analysis) in the following directions:

L1 SPLINES: The L1 splines under consideration are all locally calculated L1 splines. The focus shifted from univariate interpolating splines, which had been the topic of investigation until summer 2011, to univariate approximating splines. Two options for approximating splines were considered, namely, smoothing splines and spline fits. Smoothing splines require the user to choose a balance parameter (parameter that determines the balance between how closely the data is fit and how smooth the spline is). However, there is no theoretical or empirical guidance for how to choose the balance parameter. In contrast to smoothing splines, spline fits do not involve a balance parameter or any other parameters that the user is required to choose. For this reason, it was decided not to proceed with smoothing splines but rather to use spline fits. An algorithm for calculating univariate L1 spline fits that involves multiple minimizations of the spline functional on each iteration was created. The computational results for this algorithm indicate overall good performance of the procedure. However, we were not able to identify specific advantages vs. previously available L1 spline fits calculated using an interior-point algorithm developed previously in 2004. Moreover, the extension of this algorithm to higher dimensions will be computationally unattractive because the number of minimizations of the spline functional required is proportional to a constant, for example, 15 to the dth power for d-dimensional L1 spline fits. For this reason, we have formulated a new algorithm in which there will be only one minimization of the spline functional on each iteration. This algorithm is a steepest-descent algorithm to minimize a global data-fitting functional under a constraint implemented by a local
analysis-based interpolating-spline algorithm on 5-node windows. Comparison of these locally calculated L1 spline fits with globally calculated L1 spline fits previously reported in the literature indicates that the locally calculated spline fits preserve shape on the average slightly better than the globally calculated spline fits and are computationally more efficient because the locally-calculated-spline-fit algorithm can be parallelized.

L1 MCDA: We continued development of a new L1 “Multiple Component Detection and Analysis” (L1 MCDA) algorithm. To properly distinguish this algorithm from classical PCA (Principal Component Analysis) and robust PCAs, we changed its name from the previous name L1 PCA to L1 MCDA. L1 MCDA is a fundamental and complete reformulation of PCA in a framework exclusively based on the L1 norm. Direct connection with heavy-tailed statistics is a guiding principle. We completed design of and computational results for the 2D case and submitted a manuscript on this case. L1 MCDA is able to determine the main directions and the radial extent of 2D data from Gaussian and heavy-tailed distributions without and with patterned artificial outliers (clutter) as well as from distributions consisting of multiple superimposed Gaussian and heavy-tailed distributions without and with such outliers. Computational results indicate that 2D L1 MCDA is in nearly all cases superior in accuracy to the robust PCA of Croux and Ruiz-Gazen and to the robust PCA of Ke and Kanade and is competitive in computing time with these PCAs. While L1 MCDA is not competitive in computing time with standard PCA, it is always far superior in accuracy except for a Gaussian-distributed point cloud. The theoretical framework for 2D is generalizable to higher dimensions for general pattern recognition and the extension to 3D is currently under way. The local-parabola-fit-based algorithm of 2D was generalized to 3D but did not produce equivalently good results in 3D. Algorithms based on local medians and local linear fits have been investigates but they too have not yielded convergence in 3D similar to what was observed in 2D. The causes of this situation are under investigation and this investigation will continue in a follow-on project.

Lp Averaging with $0 < p < 1$: This topic was not foreseen in the original proposal but it turns out that it leads to a natural extension of L1 splines and L1 MCDA, so it was investigated in preparation for that work. We generated evidence that one can calculate generically combinatorially expensive Lp and lp averages, $0 < p < 1$, in polynomial time by restricting the data to come from a wide class of statistical distributions. Our approach differs from the approaches in the previous literature, which are based on a priori sparsity requirements or on accepting a local minimum as a replacement for a global minimum. The functionals by which Lp averages are calculated are not convex but are radially monotonic and the functionals by which lp averages are calculated are nearly so, which are the keys to solvability in polynomial time. Analytical results for symmetric, radially monotonic univariate distributions were created. An algorithm for univariate lp averaging was also created. Computational results for a Gaussian distribution, a class of symmetric heavy-tailed distributions and a class of asymmetric heavy-tailed distributions are presented. Many phenomena in human-based areas are increasingly known to be represented by data that have large numbers of outliers and belong to very heavy-tailed distributions. When tails of distributions are so heavy that even medians (L1 and l1 averages) do not exist, one needs to consider using lp minimization principles with $0 < p < 1$.

Technology Transfer